Some non-renormalization theorems in Curci-Ferrari model

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Abstract

In the present letter, a particular form of Slavnov-Taylor identities for the Curci-Ferrari model is deduced. This model consist of Yang-Mills theory in a particular non-linear covariant gauge, supplemented with mass terms for gluons and ghosts. It can be used as a regularization for the Yang-Mills theory preserving simple Slavnov-Taylor identities. Employing these identities two non-renormalization theorems are proved that reduce the number of independent renormalization factors from five to three. These new relations are verified by comparing to the already known three-loops renormalization factors. These relations include, as a particular case, the corresponding known identities in Yang-Mills theory in Landau gauge.

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1 Introduction

In last years important progress has been made in the comprehension of the non-perturbative sector of Yang-Mills theory and QCD. The main reason for that is the tremendous growth of Montecarlo simulations that have made possible the calculation of different non-perturbative quantities which are beyond the scope of perturbation theory.

Another consequence of the progress of lattice simulations has been the possibility to test non-perturbative analytical approximations. In this way, some succesful (semi) analytical calculations have been on reach. Probably the most important results of this kind are the Yang-Mills correlators, mainly using Schwinger-Dyson [1–13] and Non-Perturbative Renormalization Group [15, 16] equations and which have been compared quite successfully with lattice results [17–23]. These gauge-dependent objects have been calculated in a variety of gauges (see, for example, [24–27]) but most results have been obtained in Landau gauge.

The results of these correlators, moreover, indicate that analytical calculations with relatively simple approximations have a chance to be controlled not only in the ultraviolet sector of the theory but also in the infrared, at least in Landau gauge. Contrarily to what happens in perturbation theory, the strong coupling does not seem to diverge in the infrared. Even if, results depend on the scheme used and on the details of the approximations, different analyses seem to agree on the fact that Yang-Mills coupling remains bounded in the infrared. In the context of Landau gauge, the ghost-gluon coupling becomes particularly simple to analyze due to the well-known non-renormalization theorem for this specific coupling [36]. Many lattice results even seem to indicate that this coupling goes to zero in the infrared [22, 23]. On the same lines, gluon propagator do not have a massless behaviour as obtained in perturbation theory but a massive one (Schwinger-Dyson indicates generally an infinite mass [1, 2, 4, 7, 9, 10] and lattices a finite one [17, 18, 22, 23]).

This massive behaviour in the gluonic sector, with a frozen coupling in the infrared, reduce considerably infrared instabilities of Yang-Mills theory. However, the ghost sector remains massless in Landau gauge at a non-perturbative level and analytical or semi-analytical treatments must face typical difficulties of critical phenomena. In most cases where quantitative results for critical systems have been achieved some sort of renormalization group procedure has been necessary. The reason is that renormalization group equations only involve modes in a relatively small shell of momenta. This is particularly true in the Wilsonian version of the renormalization group [28–34] but in some sense is the characteristic property of different kinds of renormalization group equations. This must be contrasted with Schwinger-Dyson equations where all momenta are coupled in a significant way. One manifestation of this is that these equations require an explicit renormalization. This renormalization process is far from trivial to be implemented in a non-perturbative way. This is a serious motivation in order to look at the construction of renormalization group equations in Yang-Mills theory describing in a complete way correlation functions.

The ideal tool in this sense would be to have a Wilsonian flow equation for Yang-Mills theory. However, the standard procedure [37–44] does not preserve simple Slavnov-Taylor identities [36, 54] along the flow and an alternative formulation that preserve gauge symmetry has turned to be extremely involved [45–49]. Another renormalization group given by, Callan-Symanzik equation [50, 51] describes the flow of a field theory in term of a renormalized mass parameter that regulates completelly the theory in the infrared. It has poorer non-perturbative properties than Wilsonian flows, but is much simpler. In the Yang-Mills case, this requires a deformation of the theory in such a way that gluons and ghost acquire a mass, preserving renormalizability and reasonably simple Slavnov-Taylor identities. The simplest way to do that is offered by the Curci-Ferrari model [52, 53]. It corresponds to the Yang-Mills theory in a certain family of gauge fixings supplemented with a particular mass terms for gluons and ghosts. If the gauge and mass parameters go to zero, Curci-Ferrari model reduces to the important case of Yang-Mills theory in Landau gauge.

There is another motivation to consider the Curci-Ferrari model. The non-perturbative definition of a BRST symmetry [55, 56] has been elusive. In particular, Neuberger has constructed a lattice version of the BRST symmetry [57] but, he also showed [58] that averages of gauge invariant operators take the form in any finite lattice of a zero over zero expression. This difficulty is closely related to the Gribov problem [59] of non-perturbative gauge fixing in non abelian gauge theories. However, recently it has been proved [60] that the Curci-Ferrari model regularize this zero, gauge invariant correlators acquiring a definite expression for any non-zero mass. In this way, Yang-Mills theory can be seen as the zero limit of the Curci-Ferrari model whose BRST symmetry has a non-perturbative meaning. Moreover, the corresponding limit can be seen as the result of the renormalization group flow.

Finally, the Landau-gauge has more symmetries than Curci-Ferrari gauge and so is fixed under renormalization. However, if one takes a zero gauge parameter along the flow, ghost remain massless and Neuberger's zero is not regularized. Fortunately, the Landau gauge (where most of the gauge-fixed Montecarlo simulations have been performed) seems to be an attractive renormalization group fixed point (see, for example, [43]). If so, the zero mass limit automatically pushes the theory to the Landau gauge in the infrared.

For all these reasons, the analysis of Curci-Ferrari model is interesting. However, even if multiplicatively renormalizable, Curci-Ferrari model seemed to require five renormalization factors [61]. In fact, a claim of a reduced number of renormalization factors [62], turned out to be incorrect [61]. These renormalization factors can be taken to be those corresponding to the renormalization of gluon and ghost fields, gauge parameter, coupling constant and mass and have been calculated up to three loops. This is to be compared to usual linear covariant gauges where (without masses) three renormalization factors are necessary or to the Landau gauge where only two are required. This complicates considerably any analytical or semi-analytical analysis. In this letter, a perturbative analysis of the renormalization procedure of the Curci-Ferrari model is performed showing that, in fact, only three renormalization factors are required. Unexpectedly, some symmetries present in the model had not been fully taken into account up to now and, in fact, the mass and coupling renormalization can be expressed in terms of the other three renormalization factors. These relations are generalizations of the already known Landau gauge non-renormalizations theorems for these quantities [36, 63]. This result is only a small consequence of the high level of symmetry of the model that may be very useful in non-perturbative approximations.

The outline of the letter is the following: First, the Curci-Ferrari model and its symmetries are reviewed. Second, the corresponding Slavnov-Taylor identities and the associated consequences for the renormalization factors are deduced. Finally some conclusions and perspectives are presented.

2 The Curci-Ferrari model

The Curci Ferrari model is a modification of Yang-Mills theory that includes masses for gluons and ghost preserving a sort of BRST symmetry. It has also many other symmetries, one of them being an anti-BRST one. Here the model without matter fields in an euclidean space-time is considered, but it is straightforward to see that the results to be proved are true also including matter fields or considering a Minkowski space-time.

The variations under BRST and anti-BRST of different fields are:

$$\begin{aligned} sA^a_\mu &= D_\mu c^a, & \bar{s}A^a_\mu &= D_\mu \bar{c}^a, \\ sc^a &= -\tfrac{g}{2} f^{abc} c^b c^c, & \bar{s}c^a &= -\tfrac{1}{\xi}\partial_\mu A^a_\mu - \tfrac{g}{2} f^{abc}\bar{c}^b c^c, \\ s\bar{c}^a &= \tfrac{1}{\xi}\partial_\mu A^a_\mu - \tfrac{g}{2} f^{abc}\bar{c}^b c^c, & \bar{s}\bar{c}^a &= -\tfrac{g}{2} f^{abc}\bar{c}^b \bar{c}^c. \end{aligned} \tag{1}$$

Here, g is the gauge coupling and ξ is the gauge parameter of the model. A_μ is the gauge field, c and $\bar{c}$ are ghost and antighosts respectively.

The most general renormalizable Lagrangian with these symmetries and the space time euclidean symmetry, global color invariance and the conservation of the number of ghost take the form:

$$\mathcal{L} = \mathcal{L}_{YM} + \mathcal{L}_{GF} + \mathcal{L}_m. \tag{2}$$

Here $\mathcal{L}_{YM}$ is the Yang-Mills lagrangian:

$$\mathcal{L}_{YM} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}, \tag{3}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$. $\mathcal{L}_{GF}$ is the gauge fixing term, that includes a ghost sector. It takes the form:

$$\mathcal{L}_{GF} = \frac{1}{2}\partial_\mu \bar{c}^a D_\mu c^a + \frac{1}{2} D_\mu \bar{c}^a \partial_\mu c^a + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 - \xi \frac{g^2}{8} f^{abc} f^{ade} \bar{c}^b c^c \bar{c}^d c^e. \tag{4}$$

Finally the mass term $\mathcal{L}_m$ takes the form:

$$\mathcal{L}_m = m^2 \left(\frac{1}{2} A^a_\mu A^a_\mu + \xi \bar{c}^a c^a \right). \tag{5}$$

It is important to observe that the action has also other linearly realized symmetries [64]. First, there is the discrete symmetry where $c^a \to \bar{c}^a$ and $\bar{c}^a \to -c^a$. There are also two continuous symmetries. The first one has the infinitesimal form $c^a \to c^a + \epsilon \bar{c}^a$. The second, independent, symmetry correspond to $\bar{c}^a \to \bar{c}^a + \epsilon c^a$.

BRST and anti-BRST transformations are not nilpotent in the Curci-Ferrari model even after imposing equations of motion. However, one can deduce closed Slavnov-Taylor identities by exploiting equations of motion for ghost and anti-ghost fields. Consider the s^2 and $\bar{s}^2$ operations:

$$\begin{aligned}
s^2 A^a_\mu &= \bar{s}^2 A^a_\mu = s^2 c^a = \bar{s}^2 \bar{c}^a = 0,\\
\bar{s}^2 c^a &= -\frac{1}{\xi}\partial_\mu (D_\mu \bar{c}^a) + \frac{g}{2} f^{abc} \bar{c}^b \Big(-\frac{1}{\xi}\partial_\mu A^c_\mu - \frac{g}{2} f^{cde} \bar{c}^d c^e \Big) + \frac{g^2}{4} f^{abc} f^{bde} \bar{c}^d \bar{c}^e c^c,\\
s^2 \bar{c}^a &= \frac{1}{\xi}\partial_\mu (D_\mu c^a) - \frac{g}{2} f^{abc} \Big(\frac{1}{\xi}\partial_\mu A^b_\mu - \frac{g}{2} f^{bde} \bar{c}^d \bar{c}^e \Big) c^c - \frac{g^2}{4} f^{abc} f^{cde} \bar{c}^b c^d c^e.
\end{aligned} \tag{6}$$

Before deducing Slavnov-Taylor identities for these symmetries it is useful to consider also 'mixed' variations for different fields:

$$\begin{aligned}
\bar{s} s A^a_\mu &= D_\mu \Big(-\frac{1}{\xi}\partial_\nu A^a_\nu - \frac{g}{2} f^{abc} \bar{c}^b c^c \Big) + g f^{abc} (D_\mu \bar{c}^b) c^c,\\
\bar{s} s A^a_\mu &= -s \bar{s} A^a_\mu,\\
\bar{s} s c^a &= g f^{abc} c^b \Big(-\frac{1}{\xi}\partial_\nu A^c_\nu - \frac{g}{2} f^{cde} \bar{c}^d c^e \Big),\\
s \bar{s} c^a &= -\frac{1}{\xi}\partial_\mu D_\mu c^a - \frac{g^2}{4} f^{abc} f^{cde} \bar{c}^b c^d c^e - \frac{g}{2} f^{abc} \Big(\frac{1}{\xi}\partial_\nu A^b_\nu - \frac{g}{2} f^{bde} \bar{c}^d c^e \Big) c^c,\\
\bar{s} s \bar{c}^a &= \frac{1}{\xi}\partial_\mu D_\mu \bar{c}^a + \frac{g^2}{4} f^{abc} f^{bde} \bar{c}^d \bar{c}^e c^c + \frac{g}{2} f^{abc} \bar{c}^b \Big(-\frac{1}{\xi}\partial_\nu A^c_\nu - \frac{g}{2} f^{cde} \bar{c}^d c^e \Big),\\
s \bar{s} \bar{c}^a &= g f^{abc} \bar{c}^b \Big(\frac{1}{\xi}\partial_\nu A^c_\nu - \frac{g}{2} f^{cde} \bar{c}^d c^e \Big).
\end{aligned} \tag{7}$$

In order to deduce Slavnov-Taylor identities for these symmetries, it is necessary to introduce sources for the variations of different fields under BRST and anti-BRST symmetries. Because the operators are not nilpotent, one should include also sources for the s^2 and $\bar{s}^2$ variations. Moreover, the introduction of sources for the variations of fundamental fields under $s\bar{s}$, should be necessary a priori because variations under anti-BRST transformations are not BRST invariant as just seen. However, as will be seen, the only variation of

Field/Source	A	c	$\bar{c}$	K	$\bar{K}$	L	$\bar{L}$	M
Dimension	1	1	1	2	2	2	2	2
Ghost number	0	1	-1	-1	1	-2	2	0

Table 1

Canonical dimensionalities and ghost number of different fields and sources.

this kind that would be necessary is the one corresponding to $s\bar{s}A^a_\mu$. Even this particular variation will not be needed in the present letter where Slavnov-Taylor identities are only considered at $\bar{K}=0$. Let's consider the generating functional:

$$\begin{aligned}\exp(W[J,\chi,\bar{\chi},K,L,\bar{K},\bar{L}]) = \int D(A,c,\bar{c}) \exp\Bigg\{ \int d^dx \Big(&- \mathcal{L} + J^a_\mu A^a_\mu + \chi^a \bar{c}^a + \bar{\chi}^a c^a \\ &+ K^a_\mu s A^a_\mu + L^a s c^a + \bar{K}^a_\mu \bar{s} A^a_\mu + \bar{L}^a \bar{s}\bar{c}^a + M^a(-g/2 f^{abc}\bar{c}^b c^c) \\ &- \frac{1}{2\xi} M^a M^a + \frac{2}{\xi}\bar{L}^a L^a + \bar{K}^a_\mu K^a_\mu \Big) \Bigg\}. \end{aligned} \tag{8}$$

Notice that sources for composite operators are coupled non-linearly. As will be seen bellow this is necessary in order to preserve multiplicative renormalizability. Table 1 contains dimensionalities in $d=4$ and ghost numbers for different fields and sources. As usual, the Slavnov-Taylor identity associated to BRST symmetry is obtained by performing the change of variables in the functional integral $\varphi \to \varphi + \delta s \varphi$ (φ being all the fundamental fields and δ a constant grassmanian parameter). One obtains for $\bar{K}=0$:

$$\int d^dx \Big\langle J^a_\mu s A^a_\mu - \bar{\chi}^a s c^a - \chi^a s \bar{c}^a + \bar{L}^a s\bar{s}\bar{c}^a - \frac{g}{2} M^a f^{abc} s\big(\bar{c}^b c^c\big)\Big\rangle\Big|_{\bar{K}=0} = 0. \tag{9}$$

In order to close the equation we need expressions for $\langle s\bar{c}^a\rangle$, $\langle s\bar{s}\bar{c}^a\rangle$, and $f^{abc}\langle s(\bar{c}^b c^c)\rangle$. To obtain them one can use the equations of motion for ghost and anti-ghost fields. The ghost equation of motion is obtained by performing in the generating functional the infinitesimal change of variables $c^a(x) \to c^a(x) + \epsilon^a(x)$ giving:

$$\bar{\chi}^a + \frac{g}{2} f^{abc}\bar{c}^b M^c + g f^{abc} c^b L^c - D_\mu K^a_\mu = \Big\langle -\frac{1}{2} D_\mu \partial_\mu \bar{c}^a - \frac{1}{2}\partial_\mu D_\mu \bar{c}^a + \frac{\xi}{4} g^2 f^{abc} f^{cde} \bar{c}^b \bar{c}^d c^e \Big\rangle + m^2 \xi \bar{c}^a. \tag{10}$$

Here the same notation has been used for the average of fundamental fields and for the fields themselves. Along the same lines one obtains:

$$\chi^a + \frac{g}{2} f^{abc} M^b c^c + g f^{abc} \bar{c}^b \bar{L}^c - D_\mu \bar{K}^a_\mu = \Big\langle \frac{1}{2} D_\mu \partial_\mu c^a + \frac{1}{2}\partial_\mu D_\mu c^a - \frac{\xi}{4} g^2 f^{abc} f^{bde} c^c \bar{c}^d c^e \Big\rangle - m^2 \xi c^a. \tag{11}$$

This implies the following relations:

$$\begin{aligned}
\xi\langle\bar{s}^2 c^a\rangle &= \bar{\chi}^a + \frac{g}{2} f^{abc}\bar{c}^b M^c + g f^{abc} c^b L^c - D_\mu K^a_\mu - m^2\xi\bar{c}^a, \\
\xi\langle s^2 \bar{c}^a\rangle &= \chi^a + \frac{g}{2} f^{abc} c^b M^c + g f^{abc}\bar{c}^b \bar{L}^c - D_\mu \bar{K}^a_\mu + m^2\xi c^a, \\
-\xi\langle(s\bar{s}+\bar{s}s)\bar{c}^a\rangle &= \bar{\chi}^a + \frac{g}{2} f^{abc}\bar{c}^b M^c + g f^{abc} c^b L^c - D_\mu K^a_\mu - m^2\xi\bar{c}^a, \\
\xi\langle s\bar{s}\bar{c}^a\rangle &= 2\Big(- \partial_\mu\langle\bar{s}A^a_\mu\rangle - \bar{\chi}^a - \frac{g}{2} f^{abc}\bar{c}^b M^c - g f^{abc} c^b L^c + D_\mu K^a_\mu + m^2\xi\bar{c}^a\Big), \\
-\xi\langle(s\bar{s}+\bar{s}s)c^a\rangle &= \chi^a + \frac{g}{2} f^{abc} c^b M^c + g f^{abc}\bar{c}^b \bar{L}^c - D_\mu \bar{K}^a_\mu + m^2\xi c^a, \\
\xi\langle\bar{s}s c^a\rangle &= 2\Big(\partial_\mu\langle s A^a_\mu\rangle - \chi^a - \frac{g}{2} f^{abc} c^b M^c - g f^{abc}\bar{c}^b \bar{L}^c + D_\mu \bar{K}^a_\mu - m^2\xi c^a\Big).
\end{aligned} \tag{12}$$

Finally, the last remaining variation can be obtained noting that:

$$\Big\langle s\Big(- \frac{g}{2} f^{abc}\bar{c}^b c^c\Big)\Big\rangle = \Big\langle s\Big(\frac{1}{\xi}\partial_\mu A^a_\mu - \frac{g}{2} f^{abc}\bar{c}^b c^c - \frac{1}{\xi}\partial_\mu A^a_\mu\Big)\Big\rangle = \Big\langle s^2\bar{c}^a - \frac{1}{\xi}\partial_\mu s A^a_\mu\Big)\Big\rangle. \tag{13}$$

Before considering renormalization properties of the model, it is important to realize that under the infinitesimal linear symmetry

$$c^a \to c^a + \epsilon\bar{c}^a, \tag{14}$$

composite operators mix in the following way:

$$\begin{aligned}
& sA^a_\mu \to sA^a_\mu + \epsilon\bar{s}A^a_\mu \\
& sc^a \to sc^a - \epsilon g f^{abc}\bar{c}^b c^c \\
& s\Big(- \frac{g}{2} f^{abc}\bar{c}^b c^c\Big) \to s\Big(- \frac{g}{2} f^{abc}\bar{c}^b c^c\Big) - \epsilon\frac{g}{2} f^{abc}\bar{c}^b\bar{c}^c = \epsilon\bar{s}\bar{c}^a
\end{aligned} \tag{15}$$

all the other being invariant. This is equivalent to saying that sources for composite operators vary linearly in the following way:

$$\begin{aligned}
& K^a_\mu \to K^a_\mu + \epsilon\bar{K}^a_\mu \\
& \bar{L}^a \to \bar{L}^a + \epsilon M^a \\
& M^a \to M^a + 2\epsilon L^a
\end{aligned} \tag{16}$$

On the same lines, one deduce that under the infinitesimal symmetry

$$\bar{c}^a \to \bar{c}^a + \epsilon c^a, \tag{17}$$

sources for composite operators vary linearly also:

$$\begin{aligned}
& \bar{K}^a_\mu \to \bar{K}^a_\mu + \epsilon K^a_\mu \\
& L^a \to L^a + \epsilon M^a \\
& M^a \to M^a + 2\epsilon\bar{L}^a
\end{aligned} \tag{18}$$

Please observe that terms non-linear in sources are invariant under these transformations.

3 Non-renormalization theorems

In this section, the renormalization properties of the CF model implied by ST identities previously presented is considered. By substituting expressions (12) and (13) in (9) and expressing the results in terms of the effective action one arrives to:

$$\int d^dx \Bigg\{ \frac{\delta\Gamma}{\delta A^a_\mu}\frac{\delta\Gamma}{\delta K^a_\mu} + \frac{\delta\Gamma}{\delta c^a}\frac{\delta\Gamma}{\delta L^a} + \frac{\delta\Gamma}{\delta \bar{c}^a}\left(-\frac{1}{\xi_0}\partial_\mu A^a_\mu + \frac{\delta\Gamma}{\delta M^a}\right) \\ -\frac{1}{\xi_0}M^a\partial_\mu\left(\frac{\delta\Gamma}{\delta K^a_\mu}\right) - \frac{2}{\xi_0}\bar{L}^a\left(\partial_\mu\left(\frac{\delta\Gamma}{\delta \bar{K}^a_\mu}\right) - g_0 f^{abc}c^b L^c + D_\mu K^a_\mu + m_0^2\xi_0\bar{c}^a\right)\Bigg\}\Bigg|_{\bar{K}=0} = 0. \tag{19}$$

where sub-index is included to denote bare quantities.

In a loop expansion, suppose that all divergences have been renormalized at order n. Divergin terms that appear in the order $n+1$ in the effective action, have couplings with positive or zero dimension (see, for example, [65]). Let's call them $\Delta\Gamma_{div}$, and take an infinitesimal constant, ϵ. If one calls $\Gamma_{div} = S + \epsilon\Delta\Gamma_{div}$, then, in dimensions lower than four, the most general form for this functional at order $n+1$, takes the form:

$$\Gamma_{div}[A,c,\bar{c},K,\bar{K},L,\bar{L},M] = \hat{\Gamma}[A,c,\bar{c}] - \int d^dx \Bigg\{ \frac{2}{\xi}\bar{L}^a L^a - \frac{1}{2\xi}M^a M^a + \gamma \bar{K}^a_\mu K^a_\mu \\ + K^a_\mu \tilde{s}A^a_\mu + \bar{K}^a_\mu \tilde{\bar{s}}A^a_\mu + L^a\tilde{s}c^a + \bar{L}^a\tilde{\bar{s}}\bar{c}^a + M^a\left(-\frac{1}{\xi_0}\partial_\mu A^a_\mu + \tilde{s}\bar{c}^a\right)\Bigg\}. \tag{20}$$

where, for the moment, $\tilde{s}A^a_\mu$, $\tilde{\bar{s}}A^a_\mu$, $\tilde{s}c^a$, $\tilde{\bar{s}}\bar{c}^a$ and $\tilde{s}\bar{c}^a$ denote arbitrary operators of dimension two with the same transformations under linear symmetries as the corresponding bare expressions. Here the coefficients of $\bar{L}^a L^a$ and $M^a M^a$ are related by the linear symmetry (14).

Substituting these expression into the Slavnov-Taylor identity (19), one can analyse first the particular case corresponding to $\bar{L} = M = 0$. This gives the three relations:

$$\tilde{s}^2 A^a_\mu = 0, \tag{21}$$
$$\tilde{s}^2 c^a = 0, \tag{22}$$
$$\tilde{s}\hat{\Gamma} = 0. \tag{23}$$

Now, the most general operators of dimension two, respecting Lorentz invariance, global color invariance, symmetries (14) and (17) and nilpotecy (21,22) are

$$\begin{aligned}
\tilde{s}A^a_\mu &= Z\partial_\mu c^a + gf^{abc}A^b_\mu c^c,\\
\tilde{\bar{s}}A^a_\mu &= Z\partial_\mu \bar{c}^a + gf^{abc}A^b_\mu \bar{c}^c,\\
\tilde{s}c^a &= -\frac{g}{2}f^{abc}c^b c^c,\\
\tilde{\bar{s}}\bar{c}^a &= -\frac{g}{2}f^{abc}c^b c^c,\\
\tilde{s}\bar{c}^a &= \frac{1}{\xi_0}\partial_\mu A^a_\mu - \frac{g}{2}f^{abc}\bar{c}^b c^c.
\end{aligned} \tag{24}$$

The equation linear in M at $L = \bar{L} = K = 0$ becomes then,

$$\tilde{s}^2\bar{c}^a + m_0^2 c^a = \frac{1}{\xi}\frac{\delta\Gamma}{\delta\bar{c}^a}. \tag{25}$$

Then, imposing the two identities (25,23) one deduces that the most general expression for $\hat{\Gamma}$ takes the form $\hat{\Gamma} = \Gamma_m + \Gamma_{GF} + \Gamma_{YM}$, where:

$$\begin{aligned}
\Gamma_m &= \int d^dx \frac{m_0^2}{Z_m}\left\{\frac{1}{2Z_A}A^a_\mu A^a_\mu + \frac{\xi_0 Z_\xi}{Z_A Z_c}\bar{c}^a c^a\right\},\\
\Gamma_{GF} &= \int d^dx \left\{\frac{1}{2Z_c}\partial_\mu\bar{c}^a\tilde{D}_\mu c^a + \frac{1}{2Z_c}\tilde{D}_\mu\bar{c}^a\partial_\mu c^a + \frac{1}{2\xi_0 Z_\xi}(\partial_\mu A^a_\mu)^2 - \frac{Z_\xi\xi_0 g_0}{8Z_g Z_A Z_c^2}f^{abc}f^{ade}\bar{c}^b c^c\bar{c}^d c^e\right\},\\
\Gamma_{YM} &= \int d^dx \left\{\frac{1}{4Z_A}\tilde{F}^a_{\mu\nu}\tilde{F}^a_{\mu\nu}\right\}.
\end{aligned} \tag{26}$$

with

$$\begin{aligned}
\tilde{D}_\mu c^a &= \partial_\mu c^a + \frac{g_0}{Z_g\sqrt{Z_A}}f^{abc}A^b_\mu c^c,\\
\tilde{D}_\mu \bar{c}^a &= \partial_\mu \bar{c}^a + \frac{g_0}{Z_g\sqrt{Z_A}}f^{abc}A^b_\mu \bar{c}^c,\\
\tilde{F}^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \frac{g_0}{Z_g\sqrt{Z_A}}f^{abc}A^b_\mu A^b_\nu.
\end{aligned} \tag{27}$$

These factors can be absorbed in the following renormalizations:

$$\begin{aligned}
A^a_\mu &= \sqrt{Z_A}A^a_{R,\mu}, & c^a &= \sqrt{Z_c}c^a_R\\
\bar{c}^a &= \sqrt{Z_c}\bar{c}^a_R, & \xi_0 &= \tfrac{Z_A}{Z_\xi}\xi_R,\\
m_0^2 &= Z_m m_R^2, & g_0 &= Z_g g_R.
\end{aligned} \tag{28}$$

Previous parameters are related to those in terms of these renormalization factors by

$$\begin{aligned}
\xi &= \tfrac{1}{ZZ_c}\xi_0, & Z_\xi &= ZZ_c\\
Z_m &= \tfrac{Z^2 Z_c}{Z_A}, & g &= Z_g Z\sqrt{Z_A}g_0.
\end{aligned} \tag{29}$$

One concludes that renormalization factors are not independent. They must satisfy the non-renormalization theorem

$$Z_m = \frac{Z_\xi^2}{Z_c Z_A}. \tag{30}$$

Renormalization factors for the Curci-Ferrari model have been calculated up to three loops in the $\overline{MS}$ scheme by Gracey [66] and one can verify explicitly that these renormalization factors verify (30). In fact, in the recent article [67] this relation is observed in the three loops perturbative series but an all order argument and the associated symmetry is absent.

Now, one can consider linear terms in $\bar{L}$ in the identity (19). Terms proportional to $f^{abc}\bar{L}^a L^b c^c$ give another non-renormalization theorem:

$$Z_g = \sqrt{Z_A} Z_c Z_\xi^2. \tag{31}$$

Notice that the Landau gauge is the intersection of usual covariant linear gauges and the Curci-Ferrari gauge. Then, in the Landau gauge limit $\xi \to 0$, the covariant linear-gauge non-renormalization theorem of the gauge parameter is valid $Z_\xi \sim 1$. Relations (30) and (31) then become usual Landau gauge's non-renormalization theorems [36, 63].

Moreover, terms proportional to $\bar{L}^a \partial_\mu K_\mu^a$ implies that factor γ is compatible with multiplicative renormalizability:

$$\gamma = \frac{Z_\xi^2}{Z_c}. \tag{32}$$

4 Conclusions and perspectives

In this paper some particular form of Slavnov-Taylor identities valid for the Curci-Ferrari model have been deduced. They allow to deduce two non renormalization theorems that reduce the number of independent renormalization factors in this model from five to three. This relation has been tested in the already known $\overline{MS}$ three loops calculation of these renormalization factors. One of the two identities implies that the mass renormalization is fixed by the renormalization of dimensionless coupling in $d = 4$. This simplifies considerably the corresponding Callan-Symanzik equation, whose analysis is actually in progress [68].

Even if in this paper this particular Slavnov-Taylor identity has only been exploited in order to study renormalization properties of the Curci-Ferrari model, it can also be used to simplify the analysis of finite parts of vertices of the model, mainly in order to develop non-perturbative renormalizations schemes in Schwinger-Dyson or similar contexts.

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